MATH4010 Functional Analysis (2020-21): Homework 4. Deadline: 28 Oct 2020

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.
✠ Each answer paper must include your name and student ID.

1. Let $X = \{f \in C^b(-1,1) : f'$ exists and bounded continuous on $(-1,1)\}$. Suppose that X is endowed with the sup-norm. Using the Open Mapping Theorem or otherwise, show that X is a not Banach space.

2. Let X and Y be Banach spaces. Let $T_n : X \to Y$ be a sequence of bounded linear operators. Show that the followings are equivalent.

   (i) The sequence $(\|T_n(x)\|)$ is bounded for all $x \in X$.

   (ii) The sequence $(f(T_n x))$ is bounded for all $f \in X^*$ and for all $x \in X$.

   (iii) The sequence $(\|T_n\|)$ is bounded.

3. Let T is be linear isomorphism from a normed space X onto a normed space Y. Show that if T is a closed operator, then so is its inverse T^{-1}.

$* * *$ **End** $* * *$